Symbolic Logic Optimization and Encoding

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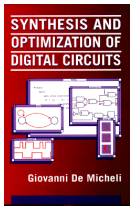


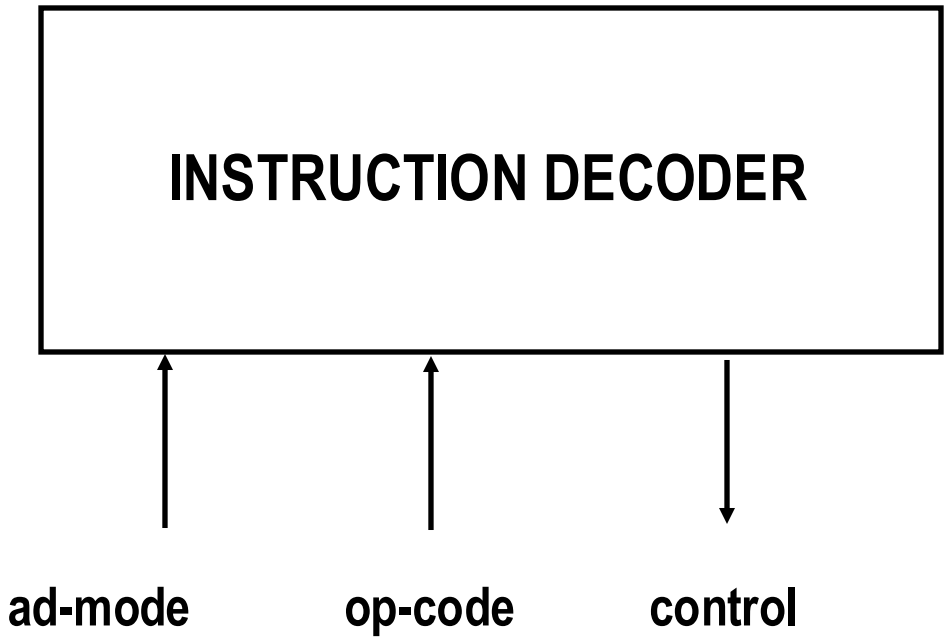
Outline

- **Symbolic minimization**
- **Encoding problems:**
  - **Input encoding**
  - **Output encoding**
  - **Mixed encoding**

Symbolic minimization

- **Minimize tables of *symbols* rather than binary tables**
  - **Extension to bvi and mvi function minimization**
- **Applications:**
  - **Simplification of tables of symbols**
  - **Simplification of interconnected logic blocks**
  - **Encoding of *finite-state machines***

Example
(input encoding)



Example

ad-mode	op-code	control
INDEX	AND	CNTA
INDEX	OR	CNTA
INDEX	JMP	CNTA
INDEX	ADD	CNTA
DIR	AND	CNTB
DIR	OR	CNTB
DIR	JMP	CNTC
DIR	ADD	CNTC
IND	AND	CNTB
IND	OR	CNTD
IND	JMP	CNTD
IND	ADD	CNTC

Definitions

- **Symbolic cover:**
  - **List of symbolic implicants**
  - **List of rows of a table**
- **Symbolic implicant:**
  - **Conjunction of symbolic literals**
- **Symbolic literals:**
  - **Simple: one symbol**
  - **Compound: the disjunction of some symbols**

INDEX	AND	CNTA
INDEX	OR	CNTA
INDEX	JMP	CNTA
INDEX	ADD	CNTA
DIR	AND	CNTB
DIR	OR	CNTB
DIR	JMP	CNTC
DIR	ADD	CNTC
IND	AND	CNTB
IND	OR	CNTD
IND	JMP	CNTD
IND	ADD	CNTC

Input encoding problem
Rationale

- Degrees of freedom in encoding the symbols
- Goal:
  - Reduce size of the representation
- Approach:
  - Encode to minimize number of rows
  - Encode to minimize number of bits

Input encoding problem

- Represent each string by 1-hot codes
- Table with positional cube notation
- Minimize table with mvi minimizer
- Interpret minimized table:
  - Compound mvi-literals
  - Groups of symbols

Example

- **Encoded cover:**

100	1000	1000
100	0100	1000
100	0010	1000
100	0001	1000
010	1000	0100
010	0100	0100
010	0010	0010
010	0001	0010
001	1000	0100
001	0100	0001
001	0010	0001
001	0001	0010

ad-mode

INDEX	100
DIR	010
IND	001

op-code

AND	1000
OR	0100
JMP	0010
ADD	0001

control

CNTA	1000
CNTB	0100
CNTC	0010
CNTD	0001

- **Minimum cover:**

100	1111	1000
010	1100	0100
001	1000	0100
010	0011	0010
001	0010	0010
001	0110	0001

Example

- **Minimum symbolic cover:**

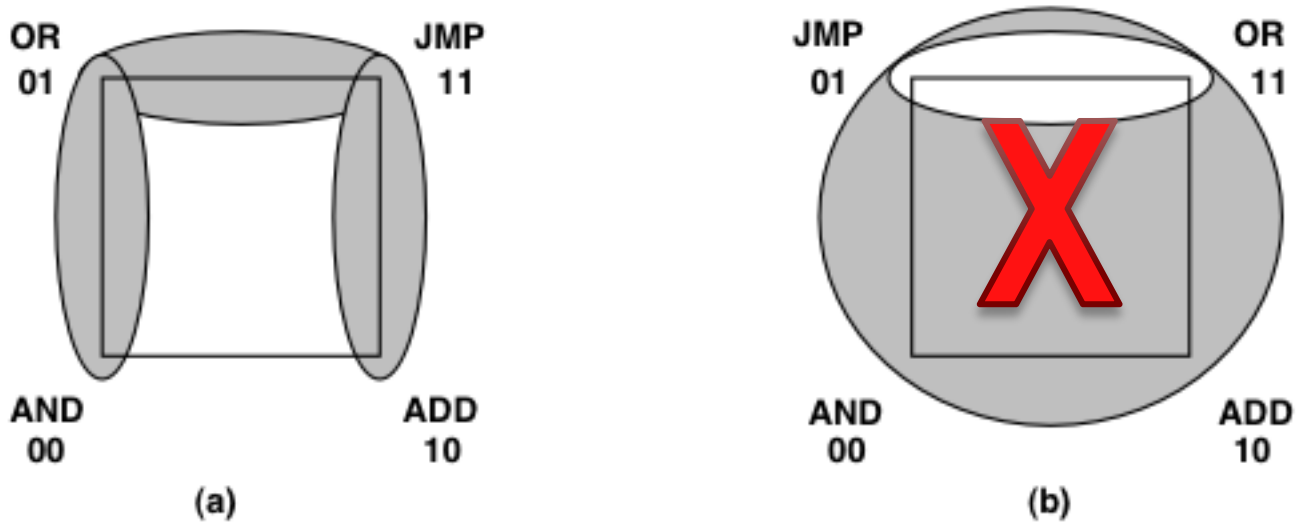
INDEX	AND,OR, JMP, ADD	CNTA
DIR	AND,OR	CNTB
IND	AND	CNTB
DIR	JMP, ADD	CNTC
IND	ADD	CNTC
IND	OR,JMP	CNTD

- **Examples of:**
  - **Simple literal: AND**
  - **Compound literal: AND, OR**

Input encoding problem

- Transform minimum symbolic cover into minimum bv-cover
- Map symbolic implicants into bv implicants (one to one)
- Compound literals:
  - Encode corresponding symbols so that their supercube does not include other symbol codes
- Replace encoded literals into cover

Example

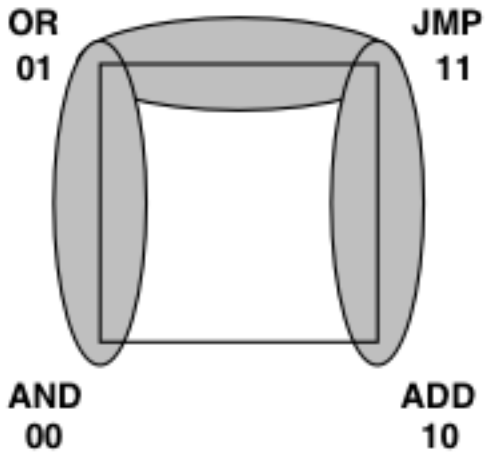


- **Compound literals:**
  - **AND, OR, JMP, ADD (similar to don't care)**
  - **AND, OR**
  - **JMP, ADD**
  - **OR, JMP**

Example

- **Valid codes:**

AND	00
OR	01
JMP	11
ADD	10



(a)

- **Replacement in cover:**

1111	→	**
1100	→	0*
1000	→	00
0011	→	1*
0010	→	10
0110	→	*1

Final cover

00	**	1000
01	0*	0100
11	00	0100
01	1*	0010
11	10	0010
11	*1	0001

Input encoding algorithms

- **Problem specification:**
  - **Constraint matrix A:**
  - **$a_{ij} = 1$ iff symbol j belongs to literal i**
- **Solution sought for:**
  - **Encoding matrix E:**
    - **As many rows as the symbols**
    - **Encoding length n_b**

Example

- **Constraint matrix:**

$$A = \begin{array}{c} \begin{matrix} \text{AND} & \text{OR} & \text{JMP} & \text{ADD} \end{matrix} \\ \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

Compound literals:

- **AND, OR, JMP, ADD = ***
- **AND, OR**
- **JMP, ADD**
- **OR, JMP**

- **Encoding matrix:**

$$E = \begin{array}{c} \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix} \\ \begin{matrix} \text{AND} \\ \text{OR} \\ \text{JMP} \\ \text{ADD} \end{matrix} \end{array}$$

Input encoding problem

- **Given constraint matrix A:**
  - **Find encoding matix E satisfying all input encoding constraints (due to compound literals)**
  - **With minimum number of columns (bits)**

Dichotomy theory

- **Dichotomy:**
  - **Two sets (*L*,*R*)**
  - **Bipartition of a subset of the symbol set**
- **Encoding:**
  - **Set of columns of E**
  - **Set of bipartitions of symbols set**
- **Rationale:**
  - **Each row of the constraint matrix implies some choice on the codes**

Dichotomies

- **Dichotomy associated with row a^T of A:**
  - **A set pair (L, R)**
    - ***L*** **has the symbols with the 1s in** a^T
    - ***R*** **has *the* symbols with the Os in** a^T
- **Seed dichotomy associated with row** a^T **of A:**
  - **A set pair (L, R)**
    - ***L*** **has the symbols with the 1s in** a^T
    - ***R*** **has one symbol with the O in** a^T

Example

- Dichotomy associated with constraint $a^T = 1100$:
  - ({AND, OR}; {JMP, ADD})
- The corresponding seed dichotomies are:
  - ({AND, OR}; {JMP})
  - ({AND, OR}; {ADD})

Definitions

- **Compatibility:**
  - **$(L_1; R_1)$ and $(L_2; R_2)$ are compatible if:**
    - $L_1 \cap R_2 = \emptyset$ and $R_1 \cap L_2 = \emptyset$
- **Covering:**
  - **Dichotomy $(L_1; R_1)$ covers $(L_2; R_2)$ if:**
    - $L_1 \supseteq L_2$ and $R_1 \supseteq R_2$
- **Prime dichotomy:**
  - **Dichotomy that is not covered by any compatible dichotomy of a given set**

Extended definitions

- **Compatibility:**
  - **$(L_1; R_1)$ and $(L_2; R_2)$ are compatible if:**
    - $L_1 \cap R_2 = \emptyset$ and $R_1 \cap L_2 = \emptyset$

      or
    - $L_1 \cap L_2 = \emptyset$ and $R_1 \cap R_2 = \emptyset$
- **Covering:**
  - **Dichotomy $(L_1; R_1)$ covers $(L_2; R_2)$ if:**
    - $L_1 \geq L_2$ and $R_1 \geq R_2$

      or
    - $L_1 \geq R_2$ and $R_1 \geq L_2$
- **Prime dichotomy:**
  - **Dichotomy that is not covered by any compatible dichotomy of a given set**

Exact input encoding

- Compute all prime dichotomies
- Form a prime/seed table
- Find minimum cover of seeds by primes

Example

- **Seed dichotomies:**

s_1	({AND, OR}	;	{JMP})
s_2	({AND, OR}	;	{ADD})
s_3	({JMP, ADD};		{AND})
s_4	({JMP, ADD};		{OR})
s_5	({OR,JMP}	;	{AND})
s_6	({OR,JMP}	;	{ADD})

- **Primes dichotomies :**

p_1	({AND, OR}	;	{JMP,ADD})
p_2	({OR,JMP}	;	{AND,ADD})
p_3	({OR, JMP, ADD}	;	{AND})
p_4	({AND, OR, JMP}	;	{ADD})

Example

- **Table:**

	s_1	s_2	s_3	s_4	s_5	s_6
p_1	1	1	1	1	0	0
p_2	0	0	0	0	1	1
p_3	0	0	1	0	1	0
p_4	0	1	0	0	0	1

- **Minimum cover : p_1 and p_2**

p_1 ({AND, OR} ; {JMP,ADD})
p_2 ({OR,JMP} ; {AND,ADD})

- **Encoding:**

$$E = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

AND OR JMP ADD

Remarks

- Satisfying all encoding constraints may require more than the minimum number of bits $Log_2\, n$ for n symbols
- 1-hot encoding satisfies all possible constraint sets, but n bits are needed
- Trade-off is possible between coding length and constraint satisfaction, that then relates to minimality of the cover

Heuristic encoding

- Determine dichotomies of rows of **A**
- Column-based encoding:
  - Construct **E** column by column
- Iterate:
  - Determine maximum compatible set of dichotomies
  - Find a compatible encoding
  - Use it as column of **E**

Example

- **Dichotomies**

d_1	({AND, OR}	;	{JMP,ADD})
d_2	({JMP,ADD}	;	{AND,OR})
d_3	({OR, JMP}	;	{AND,ADD})

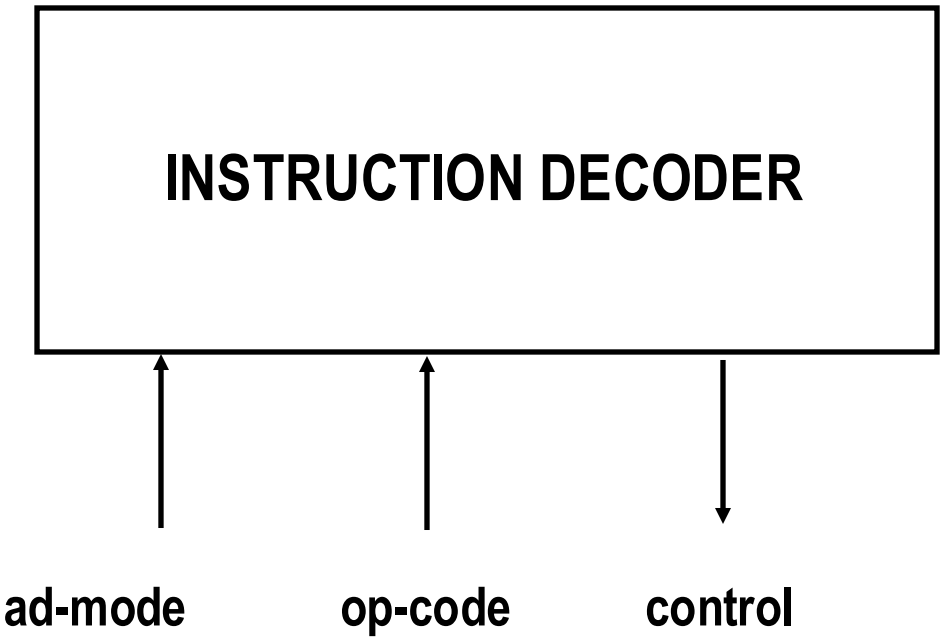
- **First two dichotomies are compatible**
- **Encoding column $[1100]^T$ satisfies d_1, d_2**
- **Need to satisfy d_3**
- **Second encoding column $[0110]^T$**

$$E = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

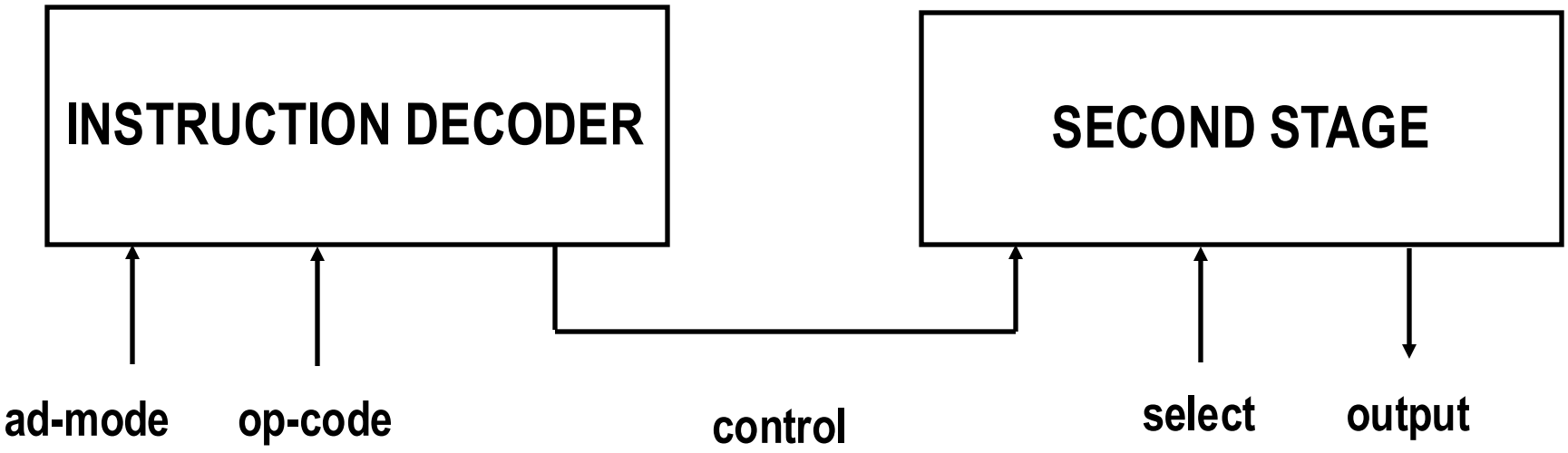
Output and mixed encoding

- **Output encoding:**
  - **Determine encoding of output symbols**
- **Mixed encoding:**
  - **Determine both input and output encoding**
  - **Examples**
    - **Interconnected circuits**
    - **Circuits with feedback**

Example



Example


INSTRUCTION DECODER
SECOND STAGE
ad-mode
op-code
control
select
output

Summary

- **Symbolic minimization:**
  - **Reduce size of tabular representations where symbols in table can be encoded**
- **Requires solving encoding problems:**
  - **Find minimum-length encoding that is valid for a minimum symbolic cover**
- **Applicable to optimizing:**
  - **Interconnected combinational blocks**
  - **Combinational part of *finite-state machines***